Class Exercise 6

Exercise 1 : Existence of Optimum Cost Paths

Let G be a graph with $\mathit{Prop}_{A*}(G)$ except that there are at most K, $K \in \mathbf{N}_0$, edges with edge cost 0. Prove that there is an optimum cost solution path for s in G if there is a solution path for s in G.

Exercise 2 : Cheapest Cost Paths

Prove the following statement:

Let be given a search graph $G = (V, E)$ with $\mathit{Prop}_{A*}(G)$ and a node set V', $V' \subseteq V$. If there is a path in G from node n, $n \in V$, to a node in V', then there is also a cheapest path in G from n to a node in V'.

Exercise 3 : Evaluation Function $f = g + h$ in A*

Let n be a node in a search space graph that is explored by A* using heuristic h.

(a) When can the value of $h(n)$ change during A*-search?

(b) When can the value of $g(n)$ change during A*-search?

Exercise 4

Give a proof (by induction) for the following statement:

Let G be a search graph with $\mathit{Prop}_{A*}(G)$. Then at any point in time before A* terminates it holds:

If $P_{s-\gamma}$ is a solution path in G, then is a shallowest OPEN node n on $P_{s-\gamma}$ and all predecessors of n on $P_{s-\gamma}$ are in CLOSED.

Exercise 5

Let G be a graph with $\mathit{Prop}_1(G)$ with edge cost values assigned, let path cost be sum of edge cost values and let h be a heuristic function for G. Answer the following questions.

(a) Does A* terminate on finite graphs?

(b) Does A* terminate on infinite graphs?

(c) Is A^* complete on finite graphs even if h is not admissible?

(d) Is A^* complete on infinite graphs even if h is not admissible?

Exercise 6 : A*: Formal Properties

Definition 1 (Constant-Bound Overestimation in h)
Let G be a search space graph with $\textit{Prop}_{A^*}(G)$ and let $b, b \in \mathbf{R}$ be some constant. A heuristic function h is called overestimating by at most b iff

$$h(n) \leq h^*(n) + b \quad \text{for all } n \in G.$$

Lemma (($C^* + b$)-Bounded OPEN Node)
Let G be a search space graph with $\textit{Prop}_{A^*}(G)$ and let A* use some heuristic function h that overestimating by at most b, $b \in \mathbf{R}$. Then, for each optimum path $P^*_{s-\gamma} \in \mathbf{P}^*_{s-\Gamma}$ and at each point in time before A* terminates there is an OPEN node n' on $P^*_{s-\gamma}$ with $f(n') \leq C^* + b$.

Give a proof for the above lemma. (Follow the steps in the proof of the C^*-Bounded OPEN Node Lemma.)