Class Exercise 7

Exercise 1 : A* Search on Trees

Prove or disprove: When A* searches a tree G, we have $g(n) = g^*(n)$ for each node in OPEN at any point in time.

Exercise 2 : Shallowest OPEN Node

Let G be a graph with $\mathit{Prop}(G)$ and let P be a path in G starting in s. Discuss the statement

> During the A* search, there is a shallowest OPEN node in P at any time before termination.

in the following situations:

- P is a finite path, but no solution path.
- P is a solution path.
- P is an infinite path.

Exercise 3 : Functions f, g, h

Let G be a graph with $\mathit{Prop}(G)$. Prove or disprove for nodes n in the traversal tree maintained by A* at any point in time:

- $g(n) \leq g^*(n)$
- $h(n) \leq h^*(n)$
- $f(n) \leq f^*(n)$

Exercise 4 : Monotonicity, Domination

Let be given two admissible (resp. monotone) heuristic functions h_1 and h_2.

Show that the heuristic functions

$$h_3(n) = \min\{h_1(n), h_2(n)\} \quad \text{and} \quad h_4(n) = \max\{h_1(n), h_2(n)\} \quad \text{and} \quad h_5(n) = \frac{1}{2}(h_1(n) + h_2(n))$$

are admissible (resp. monotone).

Let A*$_i$ denote the A* algorithm using heuristic function h_i, i=1,2,3,4,5.

Which domination relations hold for these algorithms?

Exercise 5

If A* processes a search space graph G with $\mathit{Prop}(G)$, using an admissible heuristic function h, Theorem 54 gives a necessary condition for node expansion:

> For each node n expanded by A* we have a C^*-bounded path from s to n in G.

In situations where we want to find *all* goal nodes that can be reached with cheapest cost C^*, this theorem gives both a necessary and sufficient condition for node expansion.

Give a proof of the latter statement.